CHAPTER FOUR

QUADRATIC EQUATION

Introduction:

A quadratic equation is an equation which is of the form $ax^2 + bx + c = 0$.

Examples are:

1. $2x^2 + 4x + 1 = 0$
2. $5x^2 - 4x - 2 = 0$
3. $x^2 - x + 6 = 0$

For the determination of the truth set, the roots or the solution set of a quadratic equation, a method which can be used is that of completing the squares

QUESTION 1: Find the roots or the truth set of the quadratic equation $x^2 + 5x + 6 = 0$

SOLUTION

METHOD 1

$x^2 + 5x + 6 = 0$

=> ( ) ( ) = 0

=> (*x*) (*x*) = 0

We next consider all the factors of 6 and fix each set of factors in turn into the above and in each case, multiply the contents of the two brackets.

If the answer we get is the same as the given equation, then we proceed or continue working and if not, then we must try the other sets of factors.

The set of factors of 6 are:

–1 and – 6

2 and 3

3 and 2

– 6 and – 1

Fixing of the first set of factors => i.e. (–1 and – 6) into (x) (x) = 0

=> (x – 1) (x – 6) = 0.

Multiplying them out

$\Rightarrow x^2 - 6x - x + 6 = 0$

$=> x^2 - 7x + 6 = 0$

Since $x^2 - 7x + 6 = 0$ is not the same as $x^2 + 5x + 6$, which is the given equation, then – 1 and – 6 will not work.

We then substitute the next set of factors i.e. 2 and 3 into (x) (x) = 0

=> (x + 2) (x + 3) = 0

Multiply this out =>

$x^2 + 3x + 2x + 6 = 0$

$=> x^2 + 5x + 6 = 0$

Since this is the same as the given equation, we proceed with the work.

From (x + 2) (x + 3) = 0

=> (x + 2) = 0 => x = 0 – 2

=> x = – 2, or (x + 3) = 0

=> x = 0 – 3 = -3

The truth set or roots is {x: x = – 2 or x = –3

Q[illegible]et of the equation $x^2 + 6x + 8 = 0$.

S[illegible]

$x^2 + 6x + 8 = 0$

(x) (x) = 0.

The set of factors of 8 are

(1) 1 and 8
(2) –2 and –4
(3) 8 and 1
(4) – 4 and –2
(5) 2 and 4
(6) 4 and 2
(7) –1 and – 8
(8) – 8 and –1

Substitute 1 and 8 into (x) (x) = 0

=> (x+ 1) (x+ 8) = 0,

=> $x^2 + 8x + x + 8 = 0$,

=> $x^2 + 9x + 8 = 0$.

Since $x^2 + 9x + 8 = 0$ is not the same as the given equation which is $x^2 + 6x + 8$, we try the next set of factors which are –2 and – 4,

=> (x – 2) (x – 4) = 0,

=> $x^2 – 4x – 2x + 8 = 0$,

=> $x^2 – 6x + 8 = 0$,

Which is also different from the given equation i.e.

$x^2 + 6x + 8 = 0$.

We therefore try 2 and 4,

=> $(x + 2)$ $(x+ 4) = 0$,

=> $x^2 + 4x + 2x + 8 = 0$,

=> $x^2 + 6x + 8 = 0$.

Since this is the same as the given equation

=> $(x + 2)$ $(x + 4) = 0$,

=> $x + 2 = 0$ => $x = 0 – 2$,

=> $x = – 2$, or $(x + 4) = 0$,

=> $x = 0 – 4$ => $x = – 4$.

The roots or truth set is given by $\{x: x = –2 \text{ or } x = – 4\}$.

QUESTION 3: Determine the values of *x*, which satisfy the equation $2x^2 + 8x + 6 = 0$.

SOLUTION

Since 2 is attached to the x^2, then this 2 must be attached to one of the x within the brackets.

=> $(2x)$ $(x \quad)$ = 0.

The factors of 6 are 2 and 3, 3 and 2, -2 and -3, -3 and -2, I and 6, 6 and 1, -1 and -6 as well as - 6 and -1. Now let us try the set 2 and 3 => $(2x + 2)$ $(x + 3) = 0$,

=> $2x^2 + 6x + 2x + 6 = 0$,

=> $2x^2 + 8x + 6 = 0$,

Which is the same as the given equation i.e. $2x^2 + 8x + 6 = 0$.

∴ $(2x + 2)(x + 3) = 0$,

=> $2x + 2 = 0$ => $2x = 0 - 2 = -2$.

$\therefore 2x = -2 \Rightarrow x = \frac{-2}{2} = -1$,

or $(x + 3) = 0 \Rightarrow x = 0 - 3 = -3$.

The roots or the values of x which satisfy the given equation is given by $\{x: x = -3 \text{ or } x = -1\}$

QUESTION 4: Find the truth set of the equation $22x + 10 = -4x^2$

SOLUTION

$22x + 10 = -4x^2$

$\Rightarrow 22x + 10 + 4x^2 = 0$,

$\Rightarrow 4x^2 + 22x + 10 = 0$.

Since 4 is attached to the x^2, then 4 must be attached to one of the x within the brackets.

=> (4x) (x) = 0.

The factors of 10 are 1 and 10, 2 and 5, 5 and 2, 10 and 1, -1 and -10, -10 and -1, -2 and -5, -5 and -2. Trying the first set of factors i.e. 1 and 10 => $(4x + 1)(x + 10) = 0$,

$\Rightarrow 4x^2 + 40x + x + 10 = 0$,

$\Rightarrow 4x^2 + 41x + 10 = 0$.

But this is different from $4x^2 + 22x + 10 = 0$, which is the given equation. Trying the next set i.e. 2 and 5 => $(4x + 2)(x + 5) = 0$,

$\Rightarrow 4x^2 + 20x + 2x + 10 = 0$,

$\Rightarrow 4x^2 + 22x + 10 = 0$ which is the same as the given equation. Therefore $(4x + 2)(x + 5) = 0$,

$\Rightarrow 4x + 2 = 0 \Rightarrow 4x = -2$,

$\Rightarrow x = \frac{-2}{4} = -0.5$, or $x + 5 = 0 \Rightarrow, x = -5$.

The truth set = {x: x = - 0.5 or x = -5}.

QUESTION 5: Solve the equation -3x = 10 – x^2

SOLUTION

Since -3x = 10 – x^2

=> -3x – 10 + x^2 = 0,

=> x^2 – 3x – 10 = 0.The given equation is therefore the same as $x^2 - 3x - 10 = 0$.

( ) ( ) = 0.

The factors of -10 are -5 and 2, 2 and -5, 1 and -10, -10 and 1.

Trying -5 and 2 => (x – 5) (x + 2) = 0,

=> x^2 + 2x – 5x – 10 = 0,

=> x^2 – 3x – 10 = 0,which is the same the given equation i.e $x^2 - 3x - 10 = 0$,

=> (x – 5) (x + 2) = 0,

=> x – 5 = 0, => x = 5, or x + 2 = 0, => x = -2.

∴ Truth set = {x: x = 5 or x = -2}

QUESTION 6:

Determine the truth set of $3x^2 + 14x – 5 = 0$.

SOLUTION

(3x) (x) = 0.

Factors of -5 are 1 and -5, -5 and 1, 5 and -1, -1 and 5.

Trying 1 and -5 => (3x + 1) (x – 5) = 0,

=> 3x^2 – 15x + x – 5 = 0,

=> $3x^2 - 14x - 5 = 0$ which differs from the given equation i.e. $3x^2 + 14x - 5 = 0$.

We therefore try -5 and 1

=> $(3x - 5)(x + 1) = 0$,

=> $3x^2 + 3x - 5x - 5 = 0$,

=> $3x^2 - 2x - 5 = 0$, which also differs from the given equation. We next try -1 and 5

=> $(3x - 1)(x + 5) = 0$,

=> $3x^2 + 15x - x - 5 = 0$,

$\therefore 3x^2 + 14x - 5 = 0$, which is similar to the given equation.

$\therefore (3x - 1)(x + 5) = 0$,

=> $3x - 1 = 0$ => $3x = 1$, => $x = \frac{1}{3}$

= 0.33, or $x + 5 = 0$, => $x = -5$. Truth set = $\{x: x = 0.33 \text{ or } x = 5\}$.